Acceleration of simple Harmonic oscillator

Acceleration is defined as the time rate of change of velocity

Now velocity $V = a\omega\cos(\omega t + \phi)$ ——— (i)

$\therefore$ Acceleration $\frac{dV}{dt} = Y = -a\omega^2 \sin(\omega t + \phi)$

$= a\omega^2 \sin(\omega t + \phi + \pi)$ ——— (ii)

Maximum acceleration :- The acceleration of the oscillator is maximum when $\sin(\omega t + \phi + \pi) = 1$ and is given by $\left(\frac{d^2y}{dt^2}\right)_{max} = a\omega^2$

Phase relationship between displacement, velocity and acceleration.

The velocity $V = a\omega\cos(\omega t + \phi) = a\omega \sin\left(\omega t + \phi + \frac{\pi}{2}\right)$ ——— (iii)

comparing (ii) and (iii) we find that the acceleration of a simple harmonic oscillator leads the velocity by $\pi/2$ radian in phase

The displacement $Y = a\sin(\omega t + \phi)$

comparing (ii) and (iv), we find that the acceleration of a simple harmonic oscillator leads the displacement by π radian of (180°) in phase i.e the acceleration and displacement are in antiphase.

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